

Rotational Dynamics

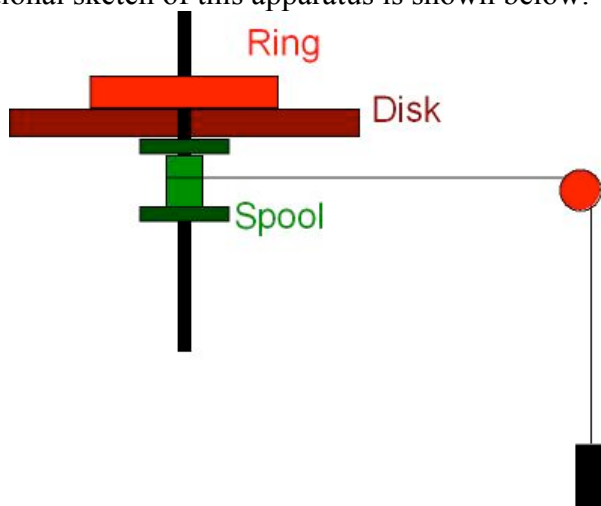
Introduction:

In this laboratory experiment we will investigate several aspects of rotational dynamics by examining torque and rotational energy considerations. In the first part of our experiment we determine the value of the rotational inertia of a solid ring and compare it with the value that we calculate using a theoretically derived expression. In the second part of the experiment we determine the speed of a hanging mass connected to our rotational apparatus by applying conservation of energy relationships for spinning objects.

The primary purpose of this lab is to become more familiar with rotational dynamics and to examine how the rotational and translational variables are related to each other. As a result, we will spend most of our attention on theoretical derivations of the relationships that we will use.

Apparatus:

The experimental apparatus that we use in this experiment is manufactured by Pasco Scientific Company (www.pasco.com). A cross sectional sketch of this apparatus is shown below:



A hanging mass, m , is connected to the rotational apparatus by a string that passes over a Smart Pulley. The pulley is connected to our DataStudio system and allows us to record the speed, v , of the falling mass m (i.e. the tangential speed of the rim of the pulley) as a function of time. The other end of the string is wound around a spool, of radius r , that is coaxial with the disk of the rotational apparatus. So, as the mass m falls, it causes the disk to spin. A ring of radius R can be placed on the disk in a groove so that it also is coaxial with the apparatus.

Theory:

1. **Torque and acceleration analysis** (consider the mass of the pulley to be negligibly small):

Let us examine the forces that cause the hanging mass to accelerate downward and the torque that causes the spinning part of the apparatus to acquire a rotational acceleration. There are two forces that

act on the hanging mass: its weight (force of the earth on m) \mathbf{mg} , and the force of the string, \mathbf{F}_T (tension in the string). Applying Newton's Second Law we can write:

$$\mathbf{mg} - \mathbf{F}_T = \mathbf{ma} \quad (\text{Eqn. 1})$$

For the rotational apparatus we need to set the total torque (about the rotational axis) equal to the rotational inertia of the apparatus, \mathbf{I} , multiplied by its angular acceleration, α . Since the only force that produces a non-zero torque about this axis is due to the tension in the string, we can write:

$$\tau_T = \mathbf{I} \alpha \quad (\text{Eqn. 2})$$

But the angular acceleration α is related to the tangential acceleration, \mathbf{a} , at the rim of the spool (i.e. the acceleration of the string) by:

$$\mathbf{a} = \mathbf{r} \alpha \quad (\text{Eqn. 3})$$

Also, by using the definition of torque, we can write:

$$\tau_T = \mathbf{F}_T \mathbf{r} \quad (\text{Eqn. 4}).$$

By substituting Eqns. 3 and 4 in 2 we obtain:

$$\mathbf{F}_T \mathbf{r} = \mathbf{I} \mathbf{a} / \mathbf{r} \quad (\text{Eqn. 5})$$

Finally, we can combine Eqns. 5 and 1 to obtain an expression for the rotational inertia of the system in terms of all the other variables:

$$\mathbf{I} = \mathbf{m} [(g/a)-1] \mathbf{r}^2 \quad (\text{Eqn. 6}).$$

So, by measuring the acceleration of the falling mass we could determine the rotational inertia of the system.

2. **Energy Considerations:**

Consider when the mass \mathbf{m} is released and its speed \mathbf{v} is determined exactly at the point where it has fallen a height \mathbf{h} . Assuming negligible frictional forces we can set the loss in the potential energy of the falling mass equal to the gain in its translational kinetic energy plus the gain in the rotational kinetic energy of the system, i.e.:

$$\mathbf{mgh} = \frac{1}{2} \mathbf{mv}^2 + \frac{1}{2} \mathbf{I}\omega^2 \quad (\text{Eqn. 6}).$$

Now, the angular speed of the rotating apparatus is related to the tangential speed at the rim of the spool (i.e. the speed of the string - i.e. the speed of the falling mass) according to:

$$\mathbf{v} = \mathbf{r} \omega \quad (\text{Eqn. 7})$$

Substituting Eqn. 7 into 6 we obtain an expression for the rotational inertia of the system as:

$$I = [(2gh/v^2) - 1] m r^2 \text{ (Eqn. 8).}$$

Procedure and analysis:

1. First make sure that the Smart Pulley is at the right height so that the string winding around the spool is perfectly horizontal (why?).
2. Keep a tension in the string and rotate the disk manually to wind the string around the spool. Again, take care for the string to remain horizontal throughout the winding process.
3. Configure your DataStudio according to: **Smart Pulley** and pick **Graph** (velocity versus time)
4. Hang your mass **m** and start taking data using the DataStudio until the mass **m** is to just hit the floor.
5. Fit a line to the velocity versus time data and record the acceleration in your data table, Table 1.
6. Now, place the ring on the disk and, for the same mass **m**, repeat steps 4 and 5 above to determine the new acceleration value. Record this also in your Table 1.
7. Repeat steps 4-6 for all the other hanging masses.
8. Use a pair of vernier calipers to measure the diameter, **2r**, of the spool. Calculate **r** and record it.
9. Use equation 6 to determine the rotational inertia of the system, once with and once without the ring, for each mass **m**. Subtract these values and calculate the rotational inertia of the ring for each trial of a mass **m**.
10. Measure the mass, **M**, of the ring, and its radius, **R**, and calculate its rotational inertia using: **$I = MR^2$** . Record this in Table 3.
11. For only one mass (your choice! See step 12, below) **m** release the system and determine its speed, **v**, when it has fallen a height **h** (your choice!). Do this once with, and a second time without, the ring resting on the disk. Record these values in Table 2.
12. Use equation 8 to determine the rotational inertia of the ring from the difference in appropriate moment of inertia values in Table 2. Record this value for the ring also in table 3.
13. Write a paragraph discussing your results.

Table 1

m(kg)	a () disk only	I () disk only	a () disk and ring	I () disk and ring	I () ring
0.050					
0.100					
0.200					
0.300					
0.500					

You should have five results for the moment of inertia of the ring listed in Table 1. Calculate the average value of the moment of inertia of the ring along with its uncertainty. $I_{\text{ring}} = \underline{\hspace{2cm}}$

Radius of Spool: $r = \underline{\hspace{2cm}}$

Mass of ring: $M = \underline{\hspace{2cm}}$

Radius of ring: $R = \underline{\hspace{2cm}}$

Table 2

m(kg)	h (m)	v () disk only	I () disk only	v () disk and ring	I () disk and ring

Table 3

Method	I () ring
Torque balance	
Theory	
Energy balance	